Recap: mean acceleration during walking

- From our previous feasibility study we derived the hypothesis that mean acceleration during walking is less than $15 \, m/s^2$.

- Step 1: Specify your working hypothesis
  Mean acceleration during walking is less 15:
  
  \[ H_1: \mu < 15 \]

- Step 2: Specify your Null hypothesis
  Thus, our null hypothesis is that mean acceleration during walking is equal or larger than 15:
  
  \[ H_0: \mu \geq 15 \]
Recap: mean acceleration during walking

- **Step 3a: Conduct experiment, collect sample data**
  - Investigate \( n \) persons
  - Record for each person the mean acceleration during walking

\[
\bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i
\]

- **Empirical standard deviation**
  
  \[
  s = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \bar{x})^2}{n - 1}}
  \]
Recap: mean acceleration during walking

- **Step 4: Select statistical test**
  - We have to test whether the empirical mean of our sample is equal or larger than the given value 15

- **Step 5: Compare whether test statistic is in line with** $H_0$

---

Recap: Student’s $t$-test

- We choose **one-sample Student’s $t$-test** since it allows to test whether the empirical mean of one sample is equal, smaller or larger than a given value $\mu_0$

- A $t$-test is based on an assumption that the **sample means are normal distributed**

- We thus assume that our data consists of independent normal distributed random variables $X_1, X_2, \ldots, X_n$ having an expected value $\mu$ and a standard deviation $\sigma$

- In our example, the one sample data $X_1, X_2, \ldots, X_n$ are the **mean acceleration magnitudes from the n persons**
Recap: Student’s $t$-test

- We have to rely on the empirical standard deviation $S$ and use the test statistic
  \[ t = \sqrt{n} \frac{\bar{X} - \mu}{S} \]

- $t$ is not normal distributed, but $t$-distributed

- The exact distribution depends on the degrees of freedom $df = n - 1 = \text{number of observations} - \text{number of estimated parameters}$

- We have to check, whether our test statistic $t$ falls outside an acceptance region

- $t$ should fall outside with a probability equal to a specified significance level

  This significance level is often chosen as 5%, in which case the acceptance region is almost, but not exactly, the interval from −2 to 2

- If $t$ falls outside the acceptance region, then we reject the null hypothesis at the chosen significance level

  The correct values for the acceptance region can be looked up as quantiles in the $t$ distribution with $n - 1$ degrees of freedom
Recap: Student’s $t$ distribution quantiles

- **Columns**: $1 - \alpha$ quantile of Student’s $t$ distribution
- **Rows**: Degrees of freedom $n$

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If we have in our example a total of 10 persons and if we set the significance level to 5%, then the value of interest is 1,833

Recap: direction of an effect

- Sometimes we have prior information on the direction of an effect

- In our example we have prior information on the direction since our null hypothesis is that average acceleration magnitude is equal or larger than 15

  $$H_0: \mu \geq 15$$

- In those cases, we may choose to reject the hypothesis only if $t$ falls in the upper tail of the distribution

- This is known as testing against a one-sided alternative
Recap: rules for rejection

- **Step 5:** Compare whether test statistic is in line with $H_0$
  - Compare $t$ with $t(1 - \alpha, n - 1) = 1 - \alpha$ quantile of student's t distribution with $n - 1$ degrees of freedom
  - Rule for rejection depends on how $H_0$ was specified:
    - $H_0: \mu = \mu_0$, reject if $t > t(1 - \frac{\alpha}{2}, n - 1)$
    - $H_0: \mu \leq \mu_0$, reject if $t > t(1 - \alpha, n - 1)$
    - $H_0: \mu \geq \mu_0$, reject if $t < -t(1 - \alpha, n - 1)$

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Recap: mean acceleration during walking

- We record acceleration data during walking from 10 persons using our SensorLog app
- In addition, we record acceleration data during running as well for another analysis
- We perform the following data preprocessing steps
  - Edit text data file
  - Import into R
  - Compute acceleration magnitude
  - Transform timestamp
  - Correct coding of categorical data
  - Split walk and run
  - Compute mean acceleration magnitude
  - Compute test statistic
Recap: edit text data file

- Remove first three header lines
- Remove sensorName column
- Remove [ and ]
- Replace , with |
- Replace value with x|y|z

Recap: import into R

- We import the data into the data frame `wr` by using the function `read.table`

```r
> wr <- read.table(header=TRUE,
                   "walk_run_10_times2.txt", sep="|

> head(wr)
statusId  x        y        z    timestamp
1        5 1.493982 4.520253 8.504205 1.426419e+12
2        5 1.340753 4.367024 8.542512 1.426419e+12
3        5 1.225831 4.213795 8.504205 1.426419e+12
4        5 1.187524 4.175488 8.580819 1.426419e+12
5        5 1.302446 4.137180 8.580819 1.426419e+12
6        5 1.379060 4.175488 8.580819 1.426419e+12
```
Recap: compute acceleration magnitude

- Compute acceleration magnitude and save it into new column

\[
> \text{wr$mag} = \sqrt{\text{wr$x}^2 + \text{wr$y}^2 + \text{wr$z}^2}
\]

```r
> head(wr)
```

```
statusId x        y        z    timestamp      mag
1        5 1.493982 4.520253 8.504205 1.426419e+12 9.746085
2        5 1.340753 4.367024 8.542512 1.426419e+12 9.687261
3        5 1.225831 4.213795 8.504205 1.426419e+12 9.569756
4        5 1.187524 4.175488 8.580819 1.426419e+12 9.616411
5        5 1.302446 4.137180 8.580819 1.426419e+12 9.614732
6        5 1.379060 4.175488 8.580819 1.426419e+12 9.641938
```
Recap: correct coding of categorical data

- statusId is coded as numeric variable by default

- However, we know that statusId is a categorical variable since it is our activity label:
  - Walking person 1
  - Running person 1
  - Walking person 2
  - Running person 2
  - ...

- We convert statusId to a categorical variable using the function factor

```r
```

Recap: split walk and run

- We create two data frames: the first one contains the walk activities and the second one the run activities

- We use the `grep` function for differentiating between walk and run labels

```r
> walk <- wr[grep("W", wr$statusId),]
> run <- wr[grep("R", wr$statusId),]
```
Recap: compute mean acceleration magnitude

We compute mean acceleration magnitude of each person:

```r
> aggregate(walk$mag, list(walk$statusId), mean)

  Group.1  x
1       W1 12.98694
2       W2 14.31158
3       W3 14.85809
4       W4 14.55913
5       W5 13.32619
6       W6 13.09754
7       W7 13.67877
8       W8 13.61826
9       W9 13.13855
10      W10 13.93779
```

```r
> walk.mean.mag <- aggregate(walk$mag, list(walk$statusId), mean)$x
> walk.mean.mag
```

Recap: compute test statistic

- Our test statistic is given by

\[
    t = \sqrt{n} \frac{\bar{X} - \mu}{S}
\]

```r
> mean(walk.mean.mag)
[1] 13.75128

> sd(walk.mean.mag)
[1] 0.6512431

> sqrt(10) * (13.75128 - 15) / 0.6512431
[1] -6.06348
```
Recap: compare whether test statistic is in line with $H_0$

- Results from our experiment
  - Average acceleration magnitude $\bar{x} = 13.75$
  - Standard deviation $s = 0.65$
  - Test statistic $t = -6.06$
- $H_0: \mu \geq 15$ can be rejected if $t < -t(1-\alpha, n-1)$

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- Since $t < -1.833$ we reject $H_0$ with a significance level $\alpha = 5\%$
  - Mean acceleration magnitude during walking is not larger than 15

Recap: Student’s $t$-test in R

- Assuming that data come from a normal distribution, we perform a one-sided Student’s $t$-test

```r
> t.test(walk.mean.mag, mu=15, alternative="less")

One Sample t-test

data:  walk.mean.mag
t = -6.0635, df = 9, p-value = 9.369e-05
alternative hypothesis: true mean is less than 15
95 percent confidence interval: 
  Inf 14.1288
sample estimates:
mean of x
13.75128
```
Recap: Student’s $t$-test in R

$t = -6.0635$, df = 9, p-value = 9.369e-05

We get the $t$ statistic, the associated degrees of freedom, and the exact p-value.

We do not need to use a table of the $t$ distribution to look up which quantiles the $t$-value can be found between.

We can immediately see that $p < 0.05$ and thus that data deviate significantly from the hypothesis that mean acceleration during walking is equal or larger than 15.

---

Recap: Student’s $t$-test in R

alternative hypothesis: true mean is less than 15

In contrast to our null hypothesis that mean acceleration during walking is equal or larger than 15, an alternative hypothesis is presented.
Recap: Student’s t-test in R

95 percent confidence interval:
   -Inf 14.1288

This is a 95% confidence interval for the true mean: the set of (hypothetical) mean values from which the data do not deviate significantly

It is based on inverting the t-test by solving for the values that cause t to lie within its acceptance region

For a 95% confidence interval, the solution is
\[ \bar{x} - t_{0.975}(f) \times \text{SEM} < \mu < \bar{x} + t_{0.975}(f) \times \text{SEM} \]

[Peter Dalgaard, 2008] Introductory Statistics with R

Recap: Student’s t-test in R

- The default Student’s t-test in R is two-sided

- In the two-sided test our null hypothesis is that the mean is equal to a given value, e.g. \( \mu_0 = 14 \)

```R
> t.test(walk.mean.mag, mu=14)

One Sample t-test

data:  walk.mean.mag
t = -1.2077, df = 9, p-value = 0.2579
alternative hypothesis: true mean is not equal to 14
95 percent confidence interval:
  13.28541 14.21715
sample estimates:
  mean of x
0.1375128
```
Two-sample Student’s $t$-test

- Used to test the hypothesis that two samples come from distributions with the same mean

- Let $\bar{x}$ denote the mean of sample 1, e.g. mean acceleration magnitude during walking

- Let $\bar{y}$ denote the mean of sample 2, e.g. mean acceleration magnitude during running

- Possible null hypotheses to test
  - $\bar{x} \leq \bar{y}$
  - $\bar{x} = \bar{y}$
  - $\bar{x} \geq \bar{y}$

Two-sample Student’s $t$-test

- Two variants depending on the hypothesis under investigation

- $t$-test for paired samples
  - Data is collected from the same set of entities/persons at different times
  - Example: From each patient the initial cholesterol level is measured and at the end of the therapy the cholesterol levels are measured again from each patient

- $t$-test for two independent samples
  - Data is collected from different sets of entities/persons
  - Example: shoe size of men and women
\textit{t-test for paired samples}

- Used in repeated measures when there is only one sample that has been tested twice

- For obtaining the test statistic, the differences between all pairs must be calculated

- The average $\bar{x} - \bar{y}$ and the standard deviation $s_D$ of those differences are used in defining the test statistic

- The degree of freedom is $n - 1$

\[ t = \frac{(\bar{x} - \bar{y}) - \omega_0}{s_D / \sqrt{n}} \]

\[ x = \frac{1}{n} \sum_{i=1}^{n} x_i \quad y = \frac{1}{n} \sum_{i=1}^{n} y_i \]

- Mean of the two samples

- Standard deviation

\[ s_D = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} ((x_i - y_i) - (x - y))^2} \]
Example: mean acceleration walking vs running

- In our experiment we have recorded acceleration data during walking and running from the same persons.

- Our working hypothesis is that mean acceleration magnitude during walking is less than during running.

- Our null hypothesis is that mean acceleration magnitude during walking is larger than during running.

- Since data was collected from the same persons, we have to apply $t$-test for paired samples.

Compute sample means

We compute mean acceleration during walking for each person:

```r
> walk.mean.mag <- aggregate(walk$mag, list(walk$statusId), mean)$x
```

```r
> walk.mean.mag
```
Compute sample means

We compute mean acceleration during running for each person:

```r
> run.mean.mag <- aggregate(run$mag, list(run$statusId), mean)$x

> run.mean.mag
```

Check for normality assumption

![Q-Q Plot for Walking](image1)

![Q-Q Plot for Running](image2)
Check for normality assumption

- We don’t observe a perfect straight line in the Q-Q plots in particular for the running activity

- We will later learn so-called non-parametric statistical hypothesis tests which do not require the normal distribution assumption and which might be better suited for our exemplary data set

- For now, we go on with two-sample Student’s t-test

Compute test statistic

$$ t = \frac{(\bar{x} - \bar{y}) - \omega_0}{s_D / \sqrt{n}} $$

```r
> walk.mean <- mean(walk.mean.mag)
> run.mean <- mean(run.mean.mag)

> walk.mean - run.mean
[1] -7.527735

> sqrt(10)*(walk.mean - run.mean)/sd(walk.mean.mag - run.mean.mag)
[1] -29.39801
```
Compute test statistic

- With \( t = -29.398, \alpha = 0.05, df = n - 1 = 9 \) we can look up in the quantile table of Student’s \( t \) distribution as we did before

<table>
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<tr>
<th>df</th>
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<td>2.228</td>
</tr>
</tbody>
</table>

- Alternatively, we can compute the p-value
  > pt(-29.398, df=9)
  [1] 1.487899e-10

Paired \( t \)-test in R

- Since our null hypothesis is that mean acceleration magnitude during walking is larger than during running, our alternative is “less”
  > t.test(walk.mean.mag, run.mean.mag, alternative="less", paired=TRUE)

      Paired t-test

data:  walk.mean.mag and run.mean.mag
t = -29.398, df = 9, p-value = 1.488e-10
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
  -Inf -7.058343
sample estimates:
mean of the differences
  -7.527735
Paired $t$-test in R

$t = -29.398$, $df = 9$, $p$-value $= 1.488e-10$

alternative hypothesis: true difference in means is less than 0

- We observe that $t$ and $p$-value are identical with the values that we have computed before.

- Since $p$-value is clearly below 0.05 we can reject the null hypothesis and accept the alternative hypothesis.

Similar to the one sample Student $t$-test we can perform a two-sided test for equality of means

```r
> t.test(walk.mean.mag, run.mean.mag, alternative="two.sided", paired=TRUE)
```

Paired $t$-test

data:  walk.mean.mag and run.mean.mag
t = -29.398, df = 9, p-value = 2.976e-10
alternative hypothesis: true difference in means is not equal to 0

95 percent confidence interval:
-8.106989 -6.948481
sample estimates:
mean of the differences
-7.527735
**Paired t-test in R**

t = -29.398, df = 9, p-value = 2.976e-10
alternative hypothesis: true difference in means is not equal to 0

- Again we observe that p-value is clearly below 0.05 and thus we can reject the null hypothesis and accept the alternative hypothesis.

- This time the alternative hypothesis is that difference in means is **not equal** to 0.

---

**t-test for independent samples**

- Used when two separate sets of independent samples are obtained.

- Examples:
  - Shoe sizes of men and women
  - Effect of a medical treatment: 50 patients are treated with the medication, the other 50 receive a placebo
  - Acceleration magnitude of 5 persons who walk and 5 other persons who run

- There are two variants of the independent samples test:
  - (1) we assume that the two variances of both samples are equal
  - (2) we don’t assume equal variance
### t-test for independent samples equal variance

- Mean of the two samples
  \[
  \bar{x} = \frac{1}{n_x} \sum_{i=1}^{n_x} x_i \quad \bar{y} = \frac{1}{n_y} \sum_{i=1}^{n_y} y_i
  \]
- Standard deviation
  \[
  s = \sqrt{\frac{(n_x - 1)s_x^2 + (n_y - 1)s_y^2}{n_x + n_y - 2}}
  \]
- Test statistic
  \[
  t = \frac{\bar{x} - \bar{y}}{s \sqrt{\frac{1}{n_x} + \frac{1}{n_y}}} \quad df = n_x + n_y - 2
  \]

### Example: mean acceleration walking vs running

- In our experiment we have recorded acceleration data during walking and running from 10 persons.

- In order to apply the t-test for independent samples to our data, we take the walking data from the first 5 persons and the running data from the remaining 5 persons.

  ```r
  > walk.mean.mag2 <- walk.mean.mag[1:5]
  
  > run.mean.mag2 <- run.mean.mag[6:10]
  ```
Example: mean acceleration walking vs running

We check whether we can assume that the variances of both samples are equal: (1) we compute the variances and (2) create a boxplot

```r
> var(walk.mean.mag2)
[1] 0.6564975

> var(run.mean.mag2)
[1] 0.06225761

> boxplot(walk.mean.mag2, run.mean.mag2, names=c("Walk2", "Run2"))
```
Example: mean acceleration walking vs running

- From the previous slides we observe that we better shouldn’t assume that the variances of both samples are equal.

- In the following two slides we apply the \( t \)-test for independent samples with equal variance anyway just to see how the outcomes looks like.

```r
> t.test(walk.mean.mag2, run.mean.mag2, 
alternative="less", paired=FALSE, var.equal = TRUE)

Two Sample t-test

data:  walk.mean.mag2 and run.mean.mag2
t = -19.9973, df = 8, p-value = 2.039e-08
alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
   -Inf -6.876858
sample estimates:
mean of x mean of y
 14.00838  21.59028
```
**t-test for independent samples equal variance**

Two Sample t-test

data:  walk.mean.mag2 and run.mean.mag2
t = -19.9973, df = 8, p-value = 2.039e-08
alternative hypothesis: true difference in means is less than 0

We observe that p-value is clearly below 0.05 and thus we can reject the null hypothesis and accept the alternative hypothesis.

**t-test for independent samples unequal variance**

- Used when the two population variances are not assumed to be equal
- This test is also known as Welch's t-test
- In comparison to the t-test for independent samples with equal variance, the differences are
  - Computation of the standard deviation of both samples
  - Degrees of freedom are calculated according to the so-called Welch-Satterthwaite equation
t-test for independent samples unequal variance

- Mean of the two samples
  \[ \bar{x} = \frac{1}{n_x} \sum_{i=1}^{n_x} x_i \quad \bar{y} = \frac{1}{n_y} \sum_{i=1}^{n_y} y_i \]

- Standard deviation
  \[ s = \sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}} \]

- Test statistic
  \[ t = \frac{\bar{x} - \bar{y}}{s} \]

---

```r
> t.test(walk.mean.mag2, run.mean.mag2, alternative="less", paired=FALSE, var.equal = FALSE)

Welch Two Sample t-test

data:  walk.mean.mag2 and run.mean.mag2
  t = -19.9973, df = 4.752, p-value = 4.524e-06
  alternative hypothesis: true difference in means is less than 0
95 percent confidence interval:
  -Inf -6.809
sample estimates:
  mean of x  mean of y
  14.00838  21.59028
```
**t-test for independent samples unequal variance**

Welch Two Sample t-test

data:  walk.mean.mag2 and run.mean.mag2
t = -19.9973, df = 4.752, p-value = 4.524e-06
alternative hypothesis: true difference in means is less than 0

- We observe that the test is now called “Welch Two Sample t-test”
- Degrees of freedom df has changed
- p-value is clearly below 0.05 and thus we can reject the null hypothesis and accept the alternative hypothesis

**Distribution-free Methods**

- Sometimes we wish to avoid making the normal distribution assumption

- **Distribution-free methods** which are also called non-parametric statistical hypothesis tests are convenient alternatives

- An alternative to t-tests is the **Wilcoxon test**
Wilcoxon test – Background

- Generally, distribution-free methods are obtained by replacing the data under investigation with corresponding order statistics.

- For the one-sample Wilcoxon test, the procedure is:
  - Subtract the theoretical $\mu_0$ from each data point.
  - Rank the differences according to their absolute numerical value, ignoring the sign.
  - Calculate the sum of the positive or negative ranks as test statistic.

Wilcoxon test – Background

- What is the meaning of the Wilcoxon test statistic?

- Let’s calculate the test statistic for a toy example.

- We consider the following data vector $x$ and $\mu_0 = 4.6$

```r
> x <- c(1:10)
> x
[1]  1  2  3  4  5  6  7  8  9 10
> mu0 <- 4.6
```
Wilcoxon test – Background

- According to the test procedure we first subtract $\mu_0$ from each data point
  
  ```r
  > x-mu0
  [1] -3.6 -2.6 -1.6 -0.6  0.4  1.4  2.4  3.4  4.4  5.4
  ```

- Second, we rank the differences according to their absolute numerical value
  
  ```r
  > rank(abs(x-mu0))
  [1]  8  6  4  2  1  3  5  7  9 10
  ```

- Third, ...

... we calculate the sum of the positive ranks as test statistic

```r
> rank.mat <- rbind(x, x-mu0, rank(abs(x-mu0)))
> rank.mat
[1,] 1.0 2.0 3.0 4.0 5.0 6.0 7.0 8.0 9.0 10.0
[2,] -3.6 -2.6 -1.6 -0.6  0.4  1.4  2.4  3.4  4.4  5.4
[3,]  8.0  6.0  4.0  2.0  1.0  3.0  5.0  7.0  9.0 10.0

> sum(rank.mat[3,rank.mat[2,] > 0])
[1] 35
```

- The meaning of the test statistic is
  - We assume a symmetric data distribution of length $n$ around $\mu_0$,
  - The test statistic corresponds to selecting each number from 1 to $n$ with probability 1/2 and calculating the sum
  - The distribution of the test statistic can be approximated by a normal distribution
Example: mean acceleration during walking

- Previously, we have used the Student’s $t$-test to test our null hypothesis that the mean is equal to a given value, e.g. $\mu_0 = 15$

One Sample t-test

data: walk.mean.mag
$t = -6.0635$, df = 9, p-value = 0.0001874
alternative hypothesis: true mean is not equal to 15
95 percent confidence interval:
13.28541 14.21715
sample estimates:
mean of x
13.75128

Example: mean acceleration during walking

- We compute the test statistic for the mean acceleration data for walking

```r
> rank.mat <- rbind(walk.mean.mag, walk.mean.mag - 15, rank(abs(walk.mean.mag - 15)))
> round(rank.mat, 1)
[1,] 13 14.3 14.6 13.1 13.7 13.6 13.1 13.9
[2,] -2 -0.7 -0.1 -1.7 -1.3 -1.4 -1.9 -1.1
[3,] 10 3.0 1.0 2.0 7.0 9.0 5.0 6.0 8.0 4.0

> sum(rank.mat[3,rank.mat[2,] > 0])
[1] 0
```

- For our data the test statistic is 0
Wilcoxon Test in R

- Similar syntax like $t$-test
  
  ```r
  > wilcox.test(walk.mean.mag, mu=15)
  
  Wilcoxon signed rank test
  
  data:  walk.mean.mag
  V = 0, p-value = 0.001953
  alternative hypothesis: true location is not equal to 15
  
  V = 0 is the test statistic that we have computed in the previous slide

  p-value is below 0.05 and thus we can reject the null hypothesis and accept the alternative hypothesis

$t$-test vs. Wilcoxon Test

- We have seen the application of the parametric Student's $t$-test as well as the non-parametric Wilcoxon Test

- If the model assumptions of the parametric test are fulfilled (e.g. data come from the normal distribution), then parametric tests are usually more efficient than non-parametric ones
Two-sample Wilcoxon test

- Non-parametric version of the two-sample $t$-test

- We prefer the Wilcoxon test if we doubt the normal distribution assumptions of the $t$-test

- Two-sample Wilcoxon test is based on
  - Replacing the two-sample data by their rank (without regard to grouping) and
  - Calculating the sum of the ranks in one group

Two-sample Wilcoxon test

- We consider sampling $n_1$ values without replacement from the numbers 1 to $n_1+n_2$

- The test statistic is the sum of ranks in the first group minus its theoretical minimum

- Test statistic is zero, if all the smallest values fall in the first group
Example: mean acceleration during walking

Previously, we’ve used the $t$-test for independent samples to test the hypothesis that two samples come from distributions with the same mean

> t.test(walk.mean.mag2, run.mean.mag2, var.equal = TRUE)

Two Sample t-test

data:  walk.mean.mag2 and run.mean.mag2
t = -19.9973, df = 8, p-value = 4.078e-08
alternative hypothesis: true difference in means is not equal to 0
95 percent confidence interval:
-8.456207 6.707586
sample estimates:
mean of x mean of y
14.00838 21.59028

Two-sample Wilcoxon test

> wilcox.test(walk.mean.mag2, run.mean.mag2)

Wilcoxon rank sum test

data:  walk.mean.mag2 and run.mean.mag2
W = 0, p-value = 0.007937
alternative hypothesis: true location shift is not equal to 0

p-value is below 0.05 and thus we can reject the null hypothesis and accept the alternative hypothesis
Matched-pairs Wilcoxon test

- Similar to t-test for paired samples, Matched-pairs Wilcoxon test is used in repeated measures when there is only one sample that has been tested twice.

- For obtaining the test statistic, first the differences between all pairs must be calculated.

- With the vector of differences, the same procedure is applied like for the one-sample Wilcoxon test.

Matched-pairs Wilcoxon test in R

```r
> wilcox.test(walk.mean.mag, run.mean.mag, paired=TRUE)

Wilcoxon signed rank test

data:  walk.mean.mag and run.mean.mag
V = 0, p-value = 0.001953
alternative hypothesis: true location shift is not equal to 0
```
Matched-pairs Wilcoxon test in R

> wilcox.test(walk.mean.mag - run.mean.mag, mu=0)

Wilcoxon signed rank test
data:  walk.mean.mag - run.mean.mag
V = 0, p-value = 0.001953
alternative hypothesis: true location is not equal to 0

Homework

- Apply the different variants of Student’s t-test to your data
- Apply the different variants of Wilcoxon tests to your data